

# BRC INSTITUTE

JEE MAINS / ADVANCE ENTRANCE EXAM.S

M.M. 300

TEST - 1

TIME - 3HRS

PHYSICS - FULL SYLLABUS

CHEMISTRY - FULL SYLLABUS

MATHEMATICS - FULL SYLLABUS

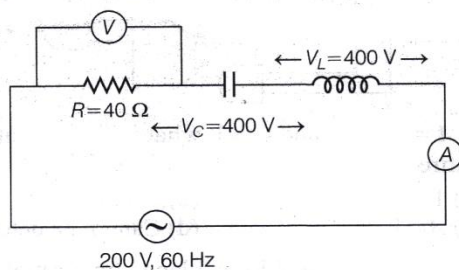
## PHYSICS

1. An electromagnetic wave propagating along the negative  $z$ -axis in vacuum has a magnetic field component at a point in space given by

$$\mathbf{B} = 3.0 \times 10^{-9} \hat{i} T$$

at a certain instant. If the frequency of the wave is 100 MHz, determine the corresponding electric field vector  $\mathbf{E}$  at that point.

- (a)  $0.9 \hat{j} \text{ V/m}$  (b)  $-0.9 \hat{j} \text{ V/m}$   
 (c)  $-9.0 \hat{j} \text{ V/m}$  (d)  $9.0 \hat{j} \text{ V/m}$
2. In a given  $L$ - $C$ - $R$  circuit, resistance of the circuit is  $40 \Omega$  as given, then the voltage across terminals of resistance and current through ammeter will be



- (a) 200 V, 5 A (b) 400 V, 5 A  
 (c) 400 V,  $5\sqrt{2}$  A (d) 200 V,  $5\sqrt{2}$  A
3. The electrostatic force on the only electron in a  $\text{Be}^{3+}$  ion (atomic number  $Z = 4$ ) placed at a distance  $r$  from the nucleus is best represented by
- (a)  $\frac{4Ke^2}{r^2} \hat{r}$  (b)  $-K \frac{e^2}{r^3} \hat{r}$   
 (c)  $-4K \frac{e^2}{r^2} \hat{r}$  (d)  $4K \frac{e^2}{r^3} \hat{r}$
4. Given below are two Statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).
- Assertion (A)** Connecting multiple batteries in

parallel does not lead to a simple addition of their voltages, instead, the total effective emf depends on the internal resistance and emf balance of the individual cells.

**Reason (R)** If two cells are connected in series and one of them is placed with reversed terminals, the net emf is reduced by the emf of the reversed cell, but their total internal resistance adds up as usual. In the light of the above statements, choose the most appropriate answer from the options given below.

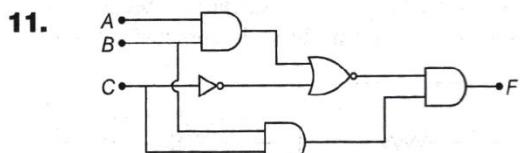
- (a) Both A and R are true, and R is the correct explanation of A.  
 (b) Both A and R are true, but R is not the correct explanation of A.  
 (c) A is true, but R is false.  
 (d) A is false, but R is true.
5. In a Young's double slit experiment, coherent monochromatic light illuminates the slits, and interference fringes are observed on a distant screen. At a certain location on the screen, the path difference between the light waves arriving from the two slits is  $\frac{5\lambda}{3}$ . What is the ratio of intensity at this location to the maximum intensity observed on the screen?
- (a)  $\frac{1}{4}$  (b)  $\frac{3}{4}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{3}$
6. A screw gauge has a pitch of 0.5 mm and 50 divisions on its circular scale. While measuring a metal wire, the main scale shows 2.5 mm and the 32nd circular division aligns with the reference line. If the length of the wire is 2.9 cm, calculate its curved surface area (in  $\text{cm}^2$ ) using correct significant figures.
- (a) 2.0 (b) 2.6 (c) 3.1 (d) 3.6
7. A body is in motion such that its kinetic energy

remains unchanged over time. Which of the following statements correctly describes the nature of the force acting on it?

- (a) The body must be moving along a straight-line path.  
 (b) The applied force, if any, acts at right angles to the direction of motion.  
 (c) The force acting on the object is always perpendicular to its acceleration vector.  
 (d) The body must be at rest if its kinetic energy is constant.
8. A device continuously supplied heat to a system at the rate of 6000 J/min. Simultaneously, the system does external work at a rate of 90 W. Calculate the time required for the internal energy of the system to increase by  $2.5 \times 10^3$  J.
- (a)  $2.4 \times 10^3$  s (b)  $2.5 \times 10^3$  s  
 (c)  $4.1 \times 10^2$  s (d)  $2.5 \times 10^2$  s
9. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).
- Assertion (A)** When a gas is rapidly compressed to half of its original volume in an insulated setup, its temperature increases.
- Reason (R)** During free expansion of an ideal gas, there is no change in internal energy and hence no change in temperature.
- In the light of the above statements, choose the correct answer from the options given below.
- (a) A is true but R is false.  
 (b) Both A and R are true but R is not the correct explanation of A.  
 (c) Both A and R are true and R is the correct explanation of A.  
 (d) A is false but R is true.
10. A deep freshwater lake has a depth of

approximately 3600 m. The bulk compressibility of water is  $49 \times 10^{-11} \text{ Pa}^{-1}$  and the density of water is  $1000 \text{ kg/m}^3$ . Estimate the fractional decrease in volume (i.e. compression) of a given amount of water due to the pressure at the bottom of the lake.

- (a)  $1.73 \times 10^{-2}$  (b)  $2.45 \times 10^{-2}$   
 (c)  $3.45 \times 10^{-2}$  (d)  $1.25 \times 10^{-2}$

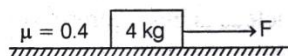


If  $A = 1, B = 0$  and  $C = 1$ , what is the output  $F$  of the entire logic circuit?

- (a) 1 (b) 0 (c) A (d) Cannot be determined

12. A block of mass 4 kg is placed on a rough horizontal surface (coefficient of friction  $\mu = 0.4$ ).

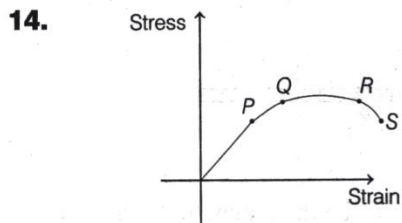
A horizontal force of 10 N is applied to the block as shown in the figure below. What is the magnitude of the frictional force acting on the block?



- (a) 16 N (b) 12 N (c) 10 N (d) 8 N

13. Two half-ring wires, each of radius  $a$ , are symmetrically placed such that together they form a full circular loop. One half (upper) carries a uniform linear charge density of  $+\sigma$  and the other half (lower) carries  $-\sigma$ . What is the magnitude of the electric field at the center of the complete loop?

- (a)  $\frac{3\sigma}{2\pi\epsilon_0 a}$  (b)  $\frac{\sigma}{\pi\epsilon_0 a}$  (c) Zero (d)  $\frac{2\sigma}{\pi\epsilon_0 a}$



Now, match the terms in Column I with corresponding terms in Column II.

Column I	Column II
A. Yield point	1. P
B. Breaking point	2. Q
C. Ultimate strength point	3. R
D. Proportionally limit point	4. S

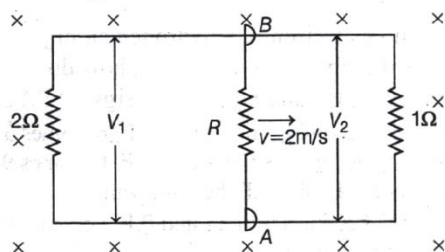
Codes

- |       |   |   |   |       |   |   |   |
|-------|---|---|---|-------|---|---|---|
| A     | B | C | D | A     | B | C | D |
| (a) 1 | 2 | 3 | 4 | (b) 2 | 1 | 3 | 4 |
| (c) 2 | 4 | 3 | 1 | (d) 1 | 4 | 3 | 2 |

15. Which one of the following statements accurately reflects a property of gravitational fields and motion under gravity?

- (a) The gravitational acceleration acting on an object varies with its own mass.  
 (b) The minimum speed required to escape Earth's gravity is unaffected by the planet's radius.  
 (c) The gravitational potential remains uniform at all points within a hollow, symmetrical spherical shell.  
 (d) A planet in orbit around the Sun maintains constant velocity throughout its path.

16. A rectangular loop has a sliding connector  $AB$  of length 1 m and resistance  $3\Omega$  and it is moving with speed 2 m/s as shown. The set-up is placed in uniform magnetic field 5 T.



Then, the values of  $V_1$  and  $V_2$  are

- (a)  $V_1 = \frac{20}{11}, V_2 = \frac{90}{11} \text{ V}$  (b)  $V_1 = \frac{20}{13}, V_2 = \frac{21}{13} \text{ V}$

- (c)  $V_1 = V_2 = \frac{20}{11} \text{ V}$  (d)  $V_1 = V_2 = \frac{21}{11} \text{ V}$

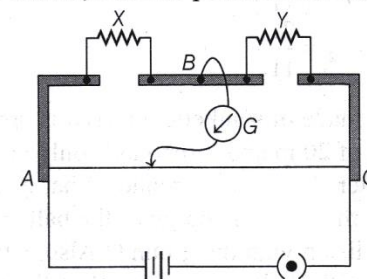
17. A ball made of synthetic material is dropped from a height of 20 m and it rebounds only up to a height of 5 m after striking the ground. What is the percentage loss in mechanical energy of the ball system due to the collision with the ground? Also, what is the speed of the ball just before it hits the ground?

- (a) 75%,  $\sqrt{(100)} \text{ m/s}$  (b) 60%,  $\sqrt{(200)} \text{ m/s}$   
 (c) 75%,  $\sqrt{(400)} \text{ m/s}$  (d) 50%,  $\sqrt{(20)} \text{ m/s}$

18. A metallic sphere's density is to be calculated. Its mass is measured as  $(24 \pm 0.5) \text{ kg}$  and its radius is recorded as  $(0.15 \pm 0.005) \text{ m}$ . What will be the approximate absolute error in the calculated value of density?

- (a)  $170 \text{ kg/m}^3$  (b)  $180 \text{ kg/m}^3$   
 (c)  $205 \text{ kg/m}^3$  (d)  $220 \text{ kg/m}^3$

19. The null point is found to be 60 cm away from the end  $A$  with resistances  $X$  and  $Y$  in position of a meter bridge as shown. When a resistance of  $15\Omega$  is connected in series with  $Y$ , then the null point is found to shift by 10 cm towards the end  $A$  of the wire. If a resistance of  $30\Omega$  wire connected in parallel with  $Y$ , then the position of null point is



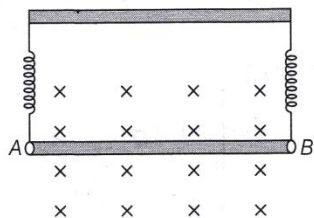
- (a) 25 cm from end  $A$  (b) 75 cm from end  $A$   
 (c) 15 cm from end  $C$  (d) 45 cm from end  $C$

20. When an electron in a hydrogen atom jumps from the 3rd to the 1st energy level, how do its kinetic energy (K.E.) and potential energy (P.E.) change?

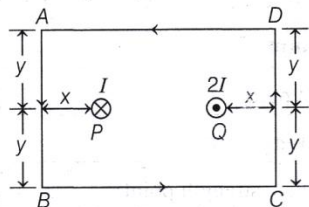
- (a) K.E. becomes 9 times and P.E. becomes 3 times
- (b) K.E. becomes 3 times and P.E. becomes 9 times
- (c) Both K.E. and P.E. become 9 times
- (d) K.E. becomes 9 times and P.E. becomes 9 times (more negative)

**NUMERICAL VALUE BASED QUESTIONS**

- 21.** A metallic wire of 50 cm length and 40 g mass is hung by two flexible cords in the presence of a uniform magnetic field of 0.40 T. Magnitude of current that must pass through the wire to make the supporting cords slack is .....  
(Take,  $g = 10 \text{ m/s}^2$ )



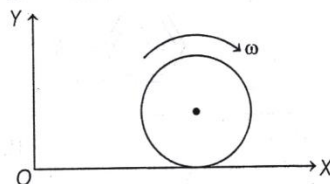
- 22.** Let  $B_P$  and  $B_Q$  be the magnetic fields produced by the wires  $P$  and  $Q$  which are placed symmetrically in a rectangular loop  $ABCD$  as shown in figure.



Current in wire  $P$  is  $I$  directed inward and in  $Q$  is  $2I$  directed outwards. If  $\int_A^B \mathbf{B}_Q \cdot d\mathbf{l} = +2\mu_0 \text{ T-m}$ ,  
 $\int_D^A \mathbf{B}_P \cdot d\mathbf{l} = -2\mu_0 \text{ T-m}$  and  $\int_A^B \mathbf{B}_P \cdot d\mathbf{l} = -\mu_0 \text{ T-m}$ ,  
then value of  $I$  will be ..... A.

- 23.** A particle undergoing simple harmonic motion has velocities of 12 cm/s and 9 cm/s, when it is at displacements of 3 cm and 4 cm from its equilibrium position, respectively. If the time period of the motion is  $\frac{2\pi}{x}$  seconds, then find the value of  $x$ .
- 24.** A disc of mass 2 kg and radius 5 cm is rolling with

angular speed of 8 rad/s on a horizontal plane as shown in figure, then the magnitude of angular momentum of the disc about the origin  $O$  is  $x \times 10^{-2} \text{ kg-m}^2/\text{s}$ . The value of  $x$  is .....



- 25.** A light ray passes through a transparent prism such that it undergoes the condition of minimum deviation. The angle of the prism is given to be  $45^\circ$  and the refractive index of the material of the prism is 1.5. The angle of incidence (in degrees) for which this minimum deviation occurs .....  
(Nearest integer).

**CHEMISTRY**

- 26.** The correct IUPAC name of  $[\text{Ag}(\text{NH}_3)_2][\text{Ag}(\text{CN})_2]$  is.  
(a) diammine silver (I) dicyanidoargentate (II)  
(b) diammine silver (I) cyanidoargentate (I)  
(c) bisamino silver (I) biscyanidoargentate (II)  
(d) diammine silver (I) dicyanidoargentate (I)
- 27.** Match the List I with List II.

List I (Molecule)		List II (Geometry)	
A.	HBr	I.	Tetrahedral
B.	$\text{H}_2\text{S}$	II.	Trigonal pyramidal
C.	$\text{NH}_3$	III.	Bent
D.	$\text{CCl}_4$	IV.	Linear

Choose the correct answer from the options given below.  
(a) A-II, B-IV, C-I, D-III (b) A-I, B-II, C-III, D-IV  
(c) A-IV, B-III, C-II, D-I (d) A-III, B-II, C-I, D-IV

- 28.** Given below are two statements.

**Assertion (A)** A process for which  $\Delta S_{\text{system}} > 0$  as

well as  $\Delta H > 0$ , passes from non-spontaneous to spontaneous state as temperature is increased.

**Reason (R)** At higher temperature  $T\Delta S$  exceeds  $\Delta H$ . In the light of the above statements choose the correct answer from the options given below.

- (a) Both A and R are true and R is the correct explanation of A.
  - (b) Both A and R are true but R is not the correct explanation of A.
  - (c) A is true, but R is false. (d) A is false, but R is true.
- 29.** If the ionisation constant of the acid is  $2.5 \times 10^{-5}$ . Then, the pH and percent dissociation of 0.08 M solution of hypochlorous acid, HOCl respectively is  
(a) 3.45, 5.00% (b) 1.76, 2.85%  
(c) 1.34, 2.45% (d) 2.85, 1.76%
- 30.** The correct statements regarding geometrical isomerism are  
A. Propene shows two-geometrical isomerism.  
B. *trans*-isomer has identical atoms groups on the opposite sides of the double bond.  
C. *cis*-but-2-ene has higher dipole moment than *trans* but-2-ene.  
D. 2-methyl but-2-ene shows two geometrical isomers.  
E. *trans*-isomer has lower melting point than *cis*-isomer.

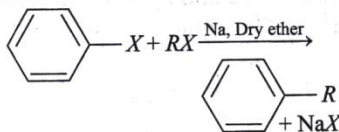
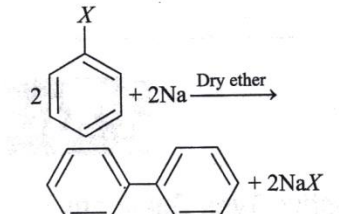
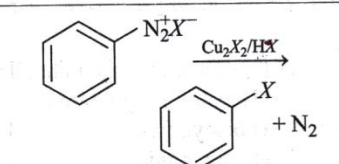
Choose the correct answer from the options given below.

- (a) A, B, C and D only (b) C, D and E only
  - (c) A, B and E only (d) B and C only
- 31.** Xenon forms several fluorides and oxofluorides which exhibit acidic behaviour. The correct sequence of descending Lewis acidity among the given species is represented by  
(a)  $\text{XeF}_6 > \text{XeOF}_4 > \text{XeF}_4 > \text{XeO}_2\text{F}_2$   
(b)  $\text{XeOF}_4 > \text{XeO}_2\text{F}_2 > \text{XeOF}_4 > \text{XeF}_6$   
(c)  $\text{XeF}_6 > \text{XeO}_2\text{F}_2 > \text{XeOF}_4 > \text{XeF}_6$   
(d)  $\text{XeF}_4 > \text{XeF}_6 > \text{XeOF}_4 > \text{XeO}_2\text{F}_2$
- 32.** 3 moles each of ethylene glycol and glucose are

dissolved in 600 g of water. The boiling point of the resultant solution is (Given, ebullioscopic constant of water = 0.52 K kg mol<sup>-1</sup>)

- (a) 378.35 K (b) 379.2 K (c) 375.3 K (d) 277.6 K

**33. Match List-I with List-II.**

List I	List II
A. 	I. Fittig reaction
B. 	II. Wurtz-Fittig reaction
C. 	III. Hunsdiecker reaction
D. $C_6H_5COOAg \xrightarrow[\Delta]{Br_2} C_6H_5Br + CO_2 + AgBr$	IV. Sandmeyer reaction

Choose the correct answer from the options given below.

- (a) A-III, B-II, C-I, D-IV (b) A-IV, B-III, C-II, D-I  
(c) A-II, B-I, C-IV, D-III (d) A-I, B-III, C-II, D-IV

**34. A zinc rod is placed in 0.1 M solution of zinc sulphate at 25°C. Assuming that the salt is dissociated to the extent of 95 percent at this dilution, then what will be the potential of the electrode at this temperature?**

$$[E^\circ_{(Zn^{2+}, Zn)} = -0.76 V]$$

- (a) +0.72 V (b) -0.79 V (c) -0.72 V (d) +0.79 V

**35. The pair of compounds having the same hybridisation for the central atom is**

- (a) XeF<sub>4</sub> and [SiF<sub>6</sub>]<sup>2-</sup>  
(b) [NiCl<sub>4</sub>]<sup>2-</sup> and [PtCl<sub>4</sub>]<sup>2-</sup>  
(c) Ni(CO)<sub>4</sub> and XeO<sub>2</sub>F<sub>2</sub>  
(d) [Co(NH<sub>3</sub>)<sub>6</sub>]<sup>3+</sup> and [Co(H<sub>2</sub>O)<sub>6</sub>]<sup>3+</sup>

**36. Given below are two statements.**

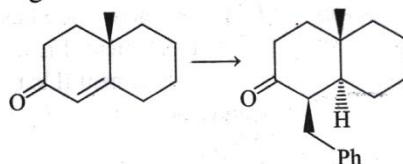
**Statement I** Nitrogen, sulphur, halogen and phosphorus present in an organic compound are detected by Lassaigne's test.

**Statement II** The elements present in the compound are converted from ionic into covalent form by fusing the compound with magnesium in Lassaigne's test.

In the light of the above statement choose the correct answer from the options given below.

- (a) Statement I is false, but statement II is true.  
(b) Both statement I and statement II are true.  
(c) Statement I is true, but statement II is false.  
(d) Both statement I and statement II are false.

**37. Which of the following is a suitable synthetic route for the given transformation?**



- (a) Na/NH<sub>3</sub>Cl; Me<sub>3</sub>SiI, Cl<sup>-</sup>/Ph-CH<sub>2</sub>-Br  
(b) (Ph-CH<sub>2</sub>)<sub>2</sub>CuLi, H<sub>3</sub>O<sup>+</sup>  
(c) Ph-CH<sub>2</sub>MgBr, H<sub>3</sub>O<sup>+</sup>  
(d) PhCH<sub>3</sub>Li, H<sub>3</sub>O<sup>+</sup>

**38. Solubility product constants (K<sub>sp</sub>) of salts of types**

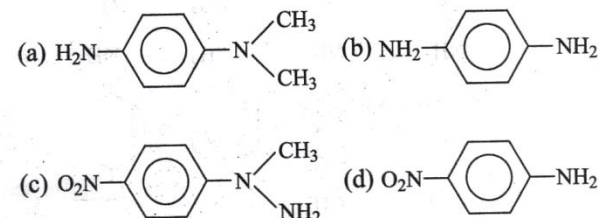
*MX*, *MX*<sub>2</sub> and *M*<sub>3</sub>*X* at temperature (*T*) are 4.0 × 10<sup>-8</sup>, 3.2 × 10<sup>-14</sup>, 2.7 × 10<sup>-15</sup> respectively solubilities (mol dm<sup>-3</sup>) of the salts at temperature '*T*' are in the order.

- (a) *MX* > *MX*<sub>2</sub> > *M*<sub>3</sub>*X* (b) *M*<sub>3</sub>*X* > *MX*<sub>2</sub> > *MX*  
(c) *MX* < *M*<sub>3</sub>*X* < *MX*<sub>2</sub> (d) *MX* > *M*<sub>3</sub>*X* > *MX*<sub>2</sub>

**39. Select the incorrect statement from the following.**

- (a) The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for resonance effect.  
(b) Tropolone is an aromatic compound and has 8πe<sup>-</sup>.  
(c) π electrons of C=O group in tropolone is not involved in aromaticity.  
(d) Benzene is more stable than hypothetical cyclohexatriene.

**40. An organic compound having molecular formula C<sub>6</sub>H<sub>4</sub>NO<sub>2</sub>F is a cyclic compound which when treated with DMF and (CH<sub>3</sub>)<sub>2</sub>NH produces *A* on further treatment with H<sub>2</sub>/Pt produces *B*. The possible molecular structure of *B* is**



- 41. For the reaction of H<sub>2</sub> and I<sub>2</sub>, the rate constant is 2.5 × 10<sup>-4</sup> dm<sup>3</sup> mol<sup>-1</sup> s<sup>-1</sup> at 327°C and 1.0 dm<sup>3</sup> mol<sup>-1</sup> s<sup>-1</sup> at 527°C. The activation energy for the reaction, in kJ mol<sup>-1</sup> is (R = 8.314 J K<sup>-1</sup> mol<sup>-1</sup>)**  
(a) 59 (b) 165 (c) 72 (d) 150

**42. Given below are two statements.**

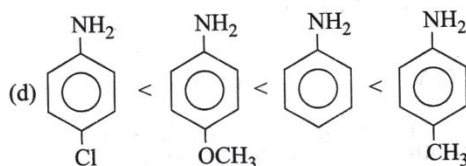
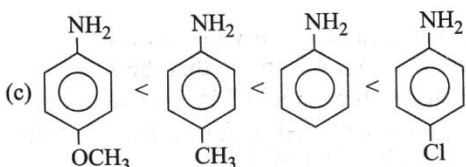
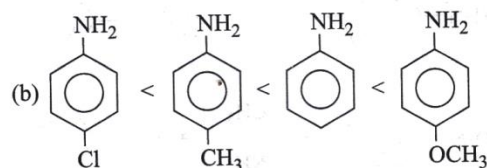
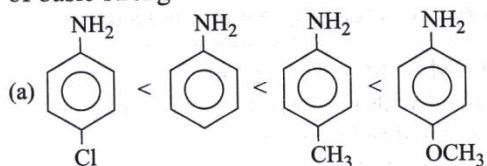
**Statement I** Invert sugar is formed during sucrose hydrolysis.

**Statement II** Natural proteins when subjected to denaturation by acids, do not convert fibrous proteins from a water soluble to a water insoluble forms.

In the light of the above statements choose the correct answer from the options given below.

- (a) Both statement I and statement II are true.  
(b) Statement I is true, but statement II is false.  
(c) Statement I is false, but statement II is true.  
(d) Both statement I and statement II are false.

43. Among the *p*-substituted anilines, the correct order of basic strength is



44. The thermite reaction used for welding of metals involves the reaction



Then the value of  $\Delta_r H^\circ$  in kJ/mol at 25°C for this reaction is —

(Given that, the standard heat of formation of  $\text{Al}_2\text{O}_3$  and  $\text{Fe}_2\text{O}_3$  are  $-1675.7 \text{ kJ mol}^{-1}$  and  $-828.4 \text{ kJ mol}^{-1}$  respectively.)

- (a)  $-347.9$  (b)  $-847.3$  (c)  $-673.4$  (d)  $-734.6$

45. Consider the following statements.

**Statement I**  $\text{Na}^+$  and  $\text{Al}^{3+}$  are isoelectronic ions but the magnitude of ionic radius of  $\text{Na}^+$  is less than that of  $\text{Al}^{3+}$ .

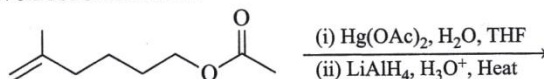
**Statement II** In isoelectronic ions, greater the negative charge lower is the ionisation energy.

In the light of the above statements, choose the correct answer from the options given below.

- (a) Both statement I and statement II are false.  
(b) Statement I is true, but statement II is false.  
(c) Statement I is false, but statement II is true.  
(d) Both statement I and statement II are true.

**NUMERICAL VALUE BASED QUESTIONS**

46. The number of molecules/ions that shows  $sp^3$ -hybridisation is/are  
 $\text{CH}_2\text{Cl}_2$ ,  $\text{NH}_4^+$ ,  $\text{SO}_4^{2-}$ ,  $\text{ClO}_4^-$ ,  $\text{NH}_3$
47. An organic compound weighing 700 mg produced 320 mg of  $\text{CO}_2$ , on complete combustion. The percentage composition of carbon in the compound is .....% (nearest integer)  
(Given molar mass in  $\text{g mol}^{-1}$  of C : 12, O : 16)
48. 8.4 g of pure aniline is treated with bromine water at room temperature to give a white precipitate of the product 'P'. The mass of product 'P' obtained is 26.4 g. The percentage yield is .....%
49. If the standard molar enthalpy change for combustion of graphite powder is  $-3.45 \times 10^2 \text{ kJ mol}^{-1}$ , the amount of heat generated on combustion of 1g of graphite powder is  $x \text{ kJ}$ . The value of  $x$  is ..... (Nearest integer)
50. The number of OH groups present in the product of given reaction is/are



**MATHEMATICS**

51.  $\mathbf{a}$  and  $\mathbf{c}$  are unit collinear vectors and  $|\mathbf{b}| = 6$ , then  $\mathbf{b} - 3\mathbf{c} = \lambda\mathbf{a}$ , if  $\lambda$  is  
(a)  $-9, 3$  (b)  $9, 3$   
(c)  $3, -3$  (d) None of these
52. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding mean are given to be 2 and 4, respectively. The variance of the combined data set is

- (a)  $5/2$  (b)  $11/2$  (c) 6 (d)  $13/2$

53. If  $f(x) = \begin{cases} (3^x - 1)^2, & x \neq 0 \\ \sin\left(\frac{x}{c}\right) \log\left(1 + \frac{x}{3}\right), & \\ 10(\log_e 3)^2, & x = 0 \end{cases}$  is continuous

at  $x = 0$ , then  $c$  is equal to

- (a)  $\frac{10}{3}$  (b)  $\frac{3}{10}$  (c)  $\frac{10}{4}$  (d)  $\frac{3}{5}$

54. Let  $f$  and  $g$  be two differentiable functions defined from  $R \rightarrow R^+$ . If  $f(x)$  has local maximum at  $x = c$  and  $g(x)$  has a local minimum at  $x = c$ , then

$$h(x) = \frac{f(x)}{g(x)}$$

- (a) has a local maximum at  $x = c$   
(b) has a local minimum at  $x = c$   
(c) is monotonic at  $x = c$   
(d) has a point of inflection at  $x = c$

55. If  $\int \frac{e^{x-1}}{(x^2 - 5x + 4)} 2x \, dx = AF(x-1) + BF(x-4) + C$

and  $F(x) = \int \frac{e^x}{x} \, dx$ , then  $(A, B)$  is equal to

- (a)  $\left(\frac{-2}{3}, \frac{8}{3}\right)$  (b)  $\left(\frac{-2}{3}, \frac{8e^3}{3}\right)$   
(c)  $\left(\frac{8}{3}, \frac{2}{3}\right)$  (d)  $\left(\frac{-2}{3}, \frac{8e^{-3}}{3}\right)$

56. The coefficient of  $x^7$  in the expansion of  $(1 - x - x^2 + x^3)^6$  is

- (a) 144 (b) 132 (c)  $-144$  (d)  $-132$

57. Evaluate  $\int x^2 e^{-2x} \, dx$

Which of the following gives the correct value (for some constant  $D$ )?

- (a)  $\frac{-1}{2} e^{-2x} \left(x^2 + x + \frac{1}{2}\right) + D$   
(b)  $\frac{-1}{2} e^{-2x} \left(x^2 - x + \frac{1}{2}\right) + D$

- (c)  $\frac{1}{2}e^{-2x} \left(x^2 + x - \frac{1}{2}\right) + D$   
 (d)  $\frac{1}{2}e^{-2x} \left(x^2 - x - \frac{1}{2}\right) + D$
- 58.** In a  $\Delta ABC$ , the vertex  $A$  is  $(1, 1)$  and orthocenter is  $(2, 4)$ . If the sides  $AB$  and  $BC$  are numbers of family of straight lines  $ax + by + c = 0$ , where  $a, b, c$  are in AP and the coordinates of vertex  $C$  are  $(\beta, \gamma)$  then the value of  $2\beta + 12\gamma$  is equal to .....
- (a) 13 (b) 14  
 (c) 16 (d) 15
- 59.** A computer producing factory has only two plants  $T_1$  and  $T_2$ . Plant  $T_1$  produces 20% and plant  $T_2$  produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective.
- It is known that  $P(\text{computer turns out to be defective, given that it is produced in plant } T_2) = 10 \times P(\text{computer turns out to be defective given that it is produced in plant } T_1)$
- A computer produced in the factory is randomly selected and it does not turn out to be defective. Then, the probability that it is produced in plant  $T_2$ , is
- (a)  $36/73$  (b)  $47/79$  (c)  $78/93$  (d)  $75/83$
- 60.** The value of 'm' for which a line with slope  $m$  is common tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  ( $a \neq b$ ) and parabola  $y^2 = 4ax$  can lie in interval is
- (a)  $(0, 1)$   
 (b)  $(-\infty, -1) \cup (1, \infty) - \left\{ \pm \sqrt{\frac{1+\sqrt{5}}{2}} \right\}$   
 (c)  $(-1, 0)$   
 (d) None of the above
- 61.** A curve passes through the point  $\left(1, \frac{\pi}{6}\right)$ . Let the slope of the curve at each point  $(x, y)$  be

- $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$ . Then, the equation of the curve is
- (a)  $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$  (b)  $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$   
 (c)  $\sec\left(\frac{2y}{x}\right) = \log x + 2$  (d)  $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$
- 62.** If  $A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 2 \\ -1 & \sqrt{3} \\ 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  and  $C = ABA^T$  and  $x = A^T C^{2023} A$ , then  $x$  is equal to
- (a)  $\begin{bmatrix} 1 & 2023 \\ 0 & 1 \end{bmatrix}$   
 (b)  $\begin{bmatrix} 4 + 2023\sqrt{3} & 6150 \\ 2025 & 4 - 2023\sqrt{3} \end{bmatrix}$   
 (c)  $\frac{1}{4} \begin{bmatrix} 2 + \sqrt{3} & 1 \\ -1 & 2 - \sqrt{3} \end{bmatrix}$   
 (d)  $\frac{1}{4} \begin{bmatrix} 2023 & 2 - \sqrt{3} \\ 2 + \sqrt{3} & 2023 \end{bmatrix}$
- 63.** Let  $d$  be the distance of the point of intersection of the lines  $\frac{x+6}{3} = \frac{y}{2} = \frac{z+1}{1}$  and  $\frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2}$  from the point  $(7, 8, 9)$ . Then,  $d^2 + 6$  is equal to
- (a) 72 (b) 69  
 (c) 75 (d) 78
- 64.** Let  $f: R \rightarrow R$  be a real differentiable function such that  $f(0) \neq 0, f(1) = 2$  and  $f(x)f(y) = f(x+y) + f(x-y)$  for all  $x, y \in R$ . Then, the value of  $\sum_{n=1}^{10} [f(n)]$  is
- (a) 20 (b) 10 (c) 30 (d) 40
- 65.** If  ${}^n P_r = {}^n P_{r+1}$  and  ${}^n C_r = {}^n C_{r-1}$ , then the value of  $r$  is

- equal to
- (a) 1 (b) 2  
 (c) 3 (d) 4
- 66.** Let  $f(x) = \lim_{n \rightarrow \infty} \tan^{-1} \left\{ 4n^2 \left( 1 - \cos \frac{x}{n} \right) \right\}$  and  $g(x) = \lim_{n \rightarrow \infty} \frac{n^2}{2} \ln \left( \cos \frac{2x}{n} \right)$ , then the least integer more than  $e^{3/4L}$ , where  $L = \lim_{x \rightarrow 0} \frac{e^{-2g(x)} - e^{f(x)}}{x^6}$ , is
- (a) 7 (b) 9  
 (c)  $\frac{8}{3}$  (d) None of these
- 67.** Consider the parabola  $y^2 = 8x$ . A focal chord  $PQ$  is drawn such that one end point is at  $(2, -4)$ . If the focus of the parabola divides the chord  $PQ$  in the ratio  $1:k$ , then the value of  $k^2 + k$  is
- (a) 5 (b) 10  
 (c) 2 (d) 13
- 68.** If the system of equations
- $$\begin{aligned} 11x + y + \lambda z &= -5 \\ 2x + 3y + 5z &= 3 \\ 8x - 19y - 39z &= \mu \end{aligned}$$
- has infinitely many solutions, then  $\lambda^4 - \mu$  is equal to
- (a) 49 (b) 45  
 (c) 47 (d) 51
- 69.** Let  $l_1, l_2, \dots, l_{100}$  be consecutive terms of an arithmetic progression with common difference  $d_1$  and let  $w_1, w_2, \dots, w_{100}$  be consecutive terms of another arithmetic progression with common difference  $d_2$ , where  $d_1 d_2 = 10$ . For each  $i = 1, 2, \dots, 100$ . Let  $R_i$  be a rectangle with length  $l_i$ , width  $w_i$  and area  $A_i$ . If  $A_{51} - A_{50} = 1000$ , then the value of  $A_{100} - A_{90}$  is
- (a) 1000 (b) 998  
 (c) 1001 (d) -1000

**70.** The 3rd, 6th and 12th terms of non-constant Arithmetic Progression (AP) are consecutive terms of a Geometric Progression (GP). If the first term of the AP is 2, find the 7th term of the AP.

- (a) 12 (b) 14  
(c) 16 (d) 18

**NUMERICAL VALUE BASED QUESTIONS**

**71.**  $\lim_{x \rightarrow 0} \left( \sum_{r=0}^n (-1)^r \cdot {}^n C_r \cdot \left( \sum_{k=0}^{n-r} {}^{n-r} C_k \cdot x^k \cdot 2^k \right) \right)$

$(x^2 - x)^r)^{\frac{1}{x}} = e^{n\lambda}$ , then  $\lambda$  is equal to .....

**72.** If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through  $(\alpha, 7, 1)$  is

$\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ , then  $\alpha$  is equal to .....

**73.** Let  $f : R \rightarrow R$  be a differentiable function with

$f(0) = 1$  and satisfying the equation

$f(x + y) = f(x)f'(y) + f'(x)f(y)$  for all  $x, y \in R$ .

Then, the value of  $\log_e (f(4))$  is equal to .....

**74.** Suppose that  $\mathbf{p}, \mathbf{q}$  and  $\mathbf{r}$  are three non-coplanar

vectors in  $R^3$ . Let the components of a vector  $\mathbf{s}$  along  $\mathbf{p}, \mathbf{q}$  and  $\mathbf{r}$  be 4, 3 and 5, respectively. If the

components of this vector  $\mathbf{s}$  along  $(-\mathbf{p} + \mathbf{q} + \mathbf{r})$ ,

$(\mathbf{p} - \mathbf{q} + \mathbf{r})$  and  $(-\mathbf{p} - \mathbf{q} + \mathbf{r})$  are  $x, y$  and  $z$

respectively, then the value of  $2x + y + z$  is equal to .....

**75.** Let  $\alpha, \beta$  be the roots of the equation  $x^2 - ax - b = 0$

with  $\text{Im}(\alpha) < \text{Im}(\beta)$ . Let  $P_n = \alpha^n - \beta^n$ . If

$P_3 = -6\sqrt{11}i, P_4 = -5\sqrt{11}i, P_5 = 9\sqrt{11}i$  and

$P_6 = 47\sqrt{11}i$ , then  $|\alpha^4 + \beta^4|$  is equal to .....